

Reflexive Ideals in Numerical Semigroups

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Mathematics

Numerical Semigroups

Definition

A set S is a **numerical semigroup** if

- 1 $S \subseteq \{0, 1, 2, 3, 4, 5, \dots\} =: \mathbb{N}$
- 2 $0 \in S$
- 3 given $a, b \in S$, then $a + b \in S$
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Example

- $S = \{0, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, \dots\}$ is a numerical semigroup
- the set of nonnegative even integers $\{0, 2, 4, 6, 8, 10, \dots\}$ is not a numerical semigroup

Chicken McNuggets Theorem

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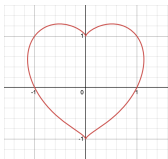
Moreover, as a special case of a Theorem of Sylvester (1814–1897), called the *Chicken McNuggets Theorem*, we have that 11 is the greatest number of nuggets you cannot buy at McDonald's.

Reflexive Ideals in Numerical Semigroups

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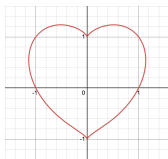
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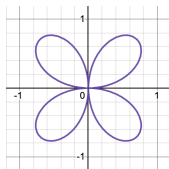
Cardioid

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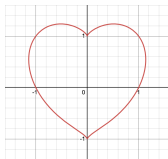
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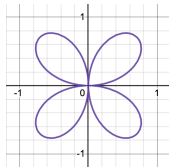
Four-leaf Clover

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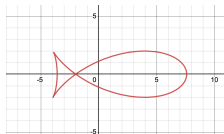
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Fish

